

## Related Rates #1

1)  $S = 4\pi r^2$

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

2)  $V = \frac{1}{3}\pi r^2 h$

$$\frac{dV}{dt} = \frac{1}{3}\pi r^2 \frac{dh}{dt} + \frac{2}{3}\pi r h \frac{dr}{dt}$$

3)  $x^2 + y^2 = z^2$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{x \frac{dx}{dt} + y \frac{dy}{dt}}{z}$$

4)  $\frac{dz}{dt} = \frac{4(0) + 3(-2)}{5}$

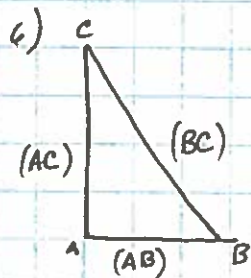
$$\frac{dz}{dt} = -\frac{6}{5}$$

5)  $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$3\pi = 4\pi(6^2) \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{48}$$



MOMENT

$$(AB) = 6 \text{ ft}$$

$$(AC) = 8 \text{ ft}$$

$$(BC) = 10 \text{ ft}$$

$$(AB)^2 + (AC)^2 = (BC)^2$$

Rates

$$\frac{d(AB)}{dt} = 2 \text{ in/sec} = \frac{1}{6} \text{ ft/sec}$$

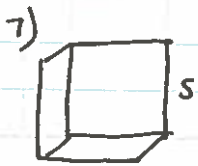
$$\frac{d(AC)}{dt} = -3 \text{ in/sec} = -\frac{1}{4} \text{ ft/sec}$$

$$\frac{d(BC)}{dt} = ?$$

$$2(AB) \frac{d(AB)}{dt} + 2(AC) \frac{d(AC)}{dt} = 2(BC) \frac{d(BC)}{dt}$$

$$\frac{d(BC)}{dt} = \frac{(AB) \frac{d(AB)}{dt} + (AC) \frac{d(AC)}{dt}}{(BC)}$$

$$= \frac{6 \text{ ft} \left( \frac{1}{6} \text{ ft/sec} \right) + 8 \text{ ft} \left( -\frac{1}{4} \text{ ft/sec} \right)}{10 \text{ ft}} = \frac{1 - 2}{10} = -\frac{1}{10} \text{ ft/sec}$$



MOMENT

$$s = 20 \text{ cm}$$

Rates

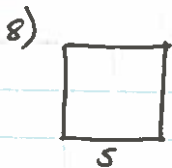
$$\frac{dV}{dt} = 1200 \text{ cm}^3/\text{min}$$

$$V = s^3$$

$$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$$

$$1200 = 3(20 \text{ cm})^2 \frac{ds}{dt}$$

$$\frac{ds}{dt} = 1 \text{ cm/min}$$



MOMENT

$$A = 16 \text{ cm}^2$$

Rates

$$\frac{ds}{dt} = 6 \text{ cm/sec}$$

$$A = s^2$$

$$\frac{dA}{dt} = 2s \frac{ds}{dt}$$

$$\frac{dA}{dt} = 2(4)(6) = 48 \text{ cm}^2/\text{sec}$$



MOMENT

$$d = 80 \text{ mm}$$

$$r = 40 \text{ mm}$$

Rates

$$\frac{dr}{dt} = -2 \text{ mm/sec}$$

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi (40)^2 (-2)$$

$$\frac{dV}{dt} = -12800\pi \text{ mm}^3/\text{sec}$$